STOCHASTIC APPROACH TO THE ESTIMATION OF RADIONUCLIDE BODY BURDENS

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It is common for estimation of radiation body burdens to use a compartment model. In these models are proposed the following simplified assumptions: 1) it is may introduced a good determined metabolism coefficients α_{ik} for each radionuclide, and 2) the law of radionuclide entering γ in organism are known in detail. Then the problem reduced to the analysis of system n independent coupling differential equations in the case of model with n compartments:

$$\frac{dq_i}{dt} = \gamma_i + \sum_{k=1}^{n} \alpha_{ik} q_k, \qquad (1)$$

where q_i is radiation loading in i-th compartment, α_{ik} are metabolism coefficients between i and k compartments and i and k are a numbers of compartments (i,k = 1,...,n). The equation (1) decided under initial conditions $q_i(t = 0) = q_i^0$. Thus the such approach to this problem is deterministic.

In reality, the situation is rather complicate. As shown the measurements, the radioactivity contamination of soil on the territory due to the accident on Chernobyl plant is inhomogeneous. This inhomogeneity is due to both the circumstances of accident and just statistical regularity of nuclide's precipitation on soil surface. Therefore, the foods (of the same kind) that are work out on this territory will to have different degree of pollution. The same result is due to the reason that transfer coefficients of radionuclides from soil to plants are not well determined parameters. In this situation the entering of activity $\gamma(t)$ in living organism is not deterministic process and rather stochastic one and in the many cases it may be approximated by impulse Poisson process:

$$\gamma(t) = \sum_j \xi_j g(t - t_j), \qquad (2)$$

where a random quantity of the impulse amplitudes ξ_j are statistic independent and characterised by a certain probability density function $w(\xi)$, and the random time moments t_j distributed on consider time interval (0,T), so that their number N is distributed according to Poison law with the parameter $\langle N \rangle = \nu T$. The function $g(\tau)$ ($g(\tau) = 0$ if $\tau < 0$) describes the form of impulse and is not random one.

Another factor that determines the stochastic of activity entering in a critical organ is temporal fluctuations in parameters α_{ik} that determine the process of metabolism. The such fluctuations may be due to random variations of organism state. In this case the dynamic of nuclides distribution on organism compartments is essentially determines by both relations between the characteristically times $t_{ik} = \alpha_{ik}^{-1}$ of nuclides transfer between compartments and times t_f, that characterise the temporary length of this fluctuations and the value of such fluctuations $\delta\alpha_{ik}$ of the metabolism parameters.

The above mentioned is shown that it allowed to speak only about the probability $W(C(t))$ that the amount of radionuclides entered in organism (organ) at moment t is equal to $C(t)$. The quantity $W(C(t))$ is very useful one since it permits to find the probabilities of radionuclide body burdens in the given interval (C_1,C_2) and probabilities of $W(C_1 < C < C_2) = \int_{C_1}^{C_2} W(C)dC$ radioactivity loading on an inhabitant above, for example, the middle loading for reference man.

Then it may to obtain in the pair correlation approximation:

$$\frac{\partial P_t(x)}{\partial t}+\sum_k\frac{\partial}{\partial x_k}\{\sum_l\alpha_{kl}x_lP_t(x)\}+\sum_i\langle\gamma_i\rangle\frac{\partial}{\partial x_i}P_t(x)=\sum_i\sum_k\frac{\partial^2}{\partial x_i\partial x_k}\{\langle\delta\gamma_i(t)\delta\gamma_k(t)\rangle P_t(x)\} \quad (3)$$

This equation describes a distribution of burdens on all n compartments. The initial conditions for equation (10) are $_{t=0}(x)=\langle\prod_{i=1}^{n}\delta(x_i-q_i^0)\rangle$, where the sign <...>denotes the average on the ensemble of the all possible realisations of the dependencies $\gamma(t)$.

The (3) is equation of Einstein-Focker-Plank type that describe the temporary evolution of burden on compartments in diffusion approximation. Since the equation (3) has coefficients that linear dependent on coordinates x_i it is may to claim that the system of random quantities x_i are obey the normal law of distribution and consequently for the unknown function P may to write down:

$$P_t(x_1,\dots,x_n)=\frac{1}{\sqrt{(2\pi)^n\det(\lambda_{jk})}}\exp\{-\frac{1}{2}\sum_{j=1}^{n}\sum_{k=1}^{n}\Lambda_{jk}(x_j-\xi_j)(x_k-\xi_k)\} \quad (4)$$

where the elements of correlation matrices $\lambda_{jk}=\lambda_{kj}$ and $\left(\Lambda_{jk}\right)=(\lambda_{jk})^{-1}$.The averages $\xi_i=\langle x_i\rangle$ and correlation coefficients λ_{jk} depend on variable t only. For determination of ξ_i and λ_{ik} we have the following equations ($\gamma_i=\langle\gamma_i\rangle(1+\delta\gamma_i)$, $<\delta\gamma_i>=0$):

$$\frac{d\xi_l}{dt}-\sum_k\alpha_{lk}\xi_k=\langle\gamma_l\rangle\,; \qquad \frac{d\lambda_{lm}}{dt}-2\sum_k(\alpha_{lk}\lambda_{km}+\alpha_{mk}\lambda_{lk})=2\langle\delta\gamma_l\delta\gamma_m\rangle,$$

Let us to consider a simple example of compartment model, namely two- compartment model that used for describing of ^{137}Cs, ^{90}Sr and ^{131}I metabolism[1]. In this case $\alpha_{12}=k_1$, $\alpha_{21}=k_2$, $\alpha_{11}=-(k_2+\lambda_p)$, $\alpha_{22}=-(k_1+k_3+\lambda_p)$ and $\gamma_1=0$, $\gamma_2=\gamma$. Here k_1 and k_2 are the coefficients of exchange by matter between compartments 1 and k_3 is the coefficient of coming out. λ_p is constant of radioactive decay. For the long-life nuclides ($\lambda_p^{-1}\geq t\geq k_i^{-1}$) we have: $\langle q_1\rangle\approx\langle\gamma\rangle k_1/k_2k_3$. For the distribution function $P(q_i)$ of burden in compartment 1 we obtained:

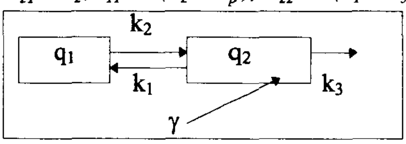

Fig.1. Two-compartment model of metabolism

$$P_t(q_1)=\left[16p^3L_{22}\det(l_{ik})\right]^{-1/2}\exp\left[-\frac{1}{2}\frac{\det(L_{ik})}{L_{22}}(q_1-\langle q_1\rangle)^2\right]\left[1+\mathrm{erf}\left\{\sqrt{L_{22}}\left[\langle q_2\rangle-\frac{L_{12}}{L_{22}}(q_1-\langle q_1\rangle)\right]\right\}\right],$$

where $rf(\eta)=\frac{2}{\sqrt{\pi}}\int_0^{\eta}e^{-t^2}dt$. In order to obtain the distribution function $P_t(q_2)$ it is sufficient to change the index 1 on 2.

Given real radioactive contamination of foods, with this method it is possible to calculate the distribution of body burdens among inhabitants. For example, knowing contamination of milk in Olevsk (Ukraine), we are calculated the distribution body burdens among inhabitats in this village. This distribution is very close to experimental data(see [2]).

In the case of temporal fluctuations in parameters of metabolism the quantity k_i it is convenient to present in the following manner:

$k_i=<k_i>(1+\delta k_i)$, $<\delta k_i>=0$,

here the sign <...> denote the average on the ensemble of the all possible realisations of the random processes γ and k_i.

The role of fluctuations in parameters k_i at the small time in the process of accumulation of radionuclides is negligable. The contribution of fluctuations k_i is essential at the time $t \geq \langle k \rangle + \min\{\alpha_{ik}\}$, that is at the times which are comparable with the times of the establishment of stationary state in considered dynamic system. We are found that the relative body burden in compartment of deposition, that is conditioned only by fluctuations of parameters k_i, is:

$$\Delta\langle q_1 \rangle = \sum_k \langle k_k \rangle \left[\frac{\sum_i \langle k_i \rangle \varepsilon_{ik} - \langle k \rangle \varepsilon_{1k}}{\langle k \rangle + \tau_{ik}} \right] \left[\langle k \rangle - \sum_i \sum_k \frac{\langle k_i \rangle \langle k_k \rangle \varepsilon_{ik}}{\langle k \rangle + \tau_{ik}} \right]^{-1} \qquad (i,k = 1,2)$$

This expression is obtained in the case of long-life radionuclides $\lambda_p << \langle k_1 \rangle + \langle k_2 \rangle + min\{\alpha_{ik}\}$. In this expression $\varepsilon_{ik} = <\delta k_i(t) \delta k_k(t)>$, and τ_{ik}^{-1} is correlation radius in cross-correlation function $<\delta k_i(t) \delta k_k(t')>$. As will readily observed, the fluctuations $\delta k_i(t)$ with small τ_{ii} has a dominant role in $\Delta\langle q_1 \rangle$. Fig. 3 gives a presentation about $\Delta\langle q_1 \rangle$ (axis z) as function of fluctuations δk_1 (axis x) and δk_3 (axis y) in the case of radionuclide I^{131} (k_1=0.04, k_3=0.09, k_2=0). Thi example is illustrated that temporal fluctuations in parametrs of metabolism effect the radiation body burdens in depot of

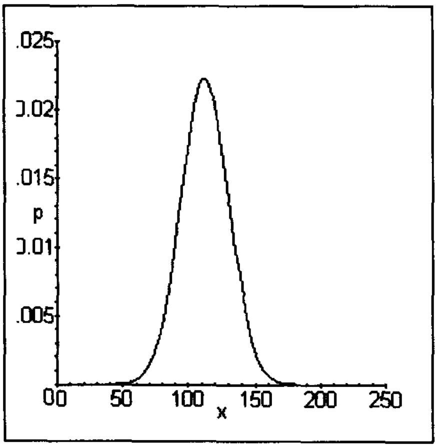

Fig.2.Distribution of body burdens among inhabitants in Olevsk.

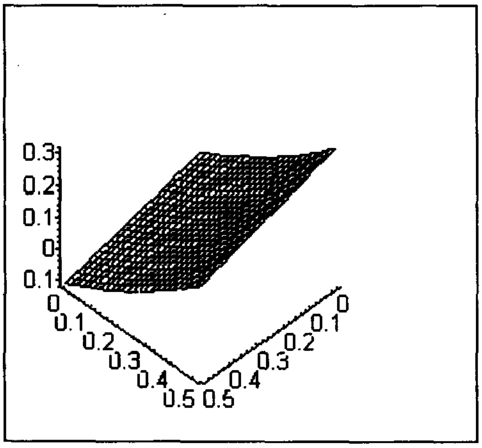

Fig.3. $\Delta\langle q_1 \rangle$ as function of fluctuations δk_1 and δk_3

radionuclides deposition. The change in body burden may be essential in the case of other nuclides (for example: ^{90}Sr, ^{45}Ca, etc.).

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