A THEORETICAL APPROACH FOR THE MEASUREMENT OF THE EFFECTIVE DOSE EQUIVALENT FOR EXTERNAL RADIATIONS

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The quantity 'effective dose equivalent', H, was introduced by the ICRP [1] to express the maximum admissible dose limits for radiation induced stochastic effects. In this paper, the relation between the response of different types of detectors and H is given explicitly, and construction requirements are deduced. Criteria for 'operational quantities' facilitating the measurements are given. It is found that the 'dose equivalent index' [1,2] is not consistent with these criteria.

THE THEORETICAL APPROACH

An irradiation situation is considered in which an upright standing antropomorphous phantom is irradiated from a spacial distribution of radiation sources located in the horizontal plane, far away from the position of the phantom. The radiation fluence ϕ at the position of the phantom in receptor-free condition [2] can then be expressed in terms of the spectral angular fluence by

$$\phi = \int_0^\infty dE \int_{2\pi} d\alpha \cdot \phi_{E,\alpha}(\alpha, E) \tag{1}$$

where the angle α indicates the direction and E the energy of the radiation. The spacial orientation of the phantom (its 'nose') is indicated by the angle α^H. Then the effective dose equivalent can be written.

$$H(\alpha^H) = \int_0^\infty dE \int_{2\pi} d\alpha \cdot M^H(\alpha - \alpha^H, E) \cdot \phi_{E,\alpha}(\alpha, E) \tag{2}$$

where the function M^H can in principle be calculated by Monte Carlo calculations and may therefore be considered as a known quantity.

For practical measurements, the angular and the energy range are decomposed in intervals $[\alpha_j, \alpha_j + \Delta\alpha_j]$ and $[E_k, E_k + \Delta E_k]$. With the definition

$$M^H_{j,k}(\alpha^H) = M^H(\alpha_j - \alpha^H, E_k) \tag{3}$$

one can write approximately

$$H(\alpha^H) \approx \sum_{j=1}^{j_m} \sum_{k=1}^{k_m} \int_{\Delta E_k} dE \int_{\Delta\alpha_j} d\alpha \cdot M^H_{j,k}(\alpha^H) \cdot g_{j,k}(\alpha, E) \cdot \phi_{E,\alpha}(\alpha, E) \tag{4}$$

where the functions g are derived by interpolating $M^H(\alpha-\alpha^H,E)/M^H_{j,k}$ within the given interval. Defining the quantity

$$\phi^H_{j,k} = \int_{\Delta E_k} dE \int_{\Delta\alpha_j} d\alpha \cdot g_{j,k}(\alpha,E) \cdot \phi_{E,\alpha}(\alpha,E) \qquad (5)$$

one can write

$$H(\alpha^H) = \sum_{j=1}^{j_m} \sum_{k=1}^{k_m} M^H_{j,k}(\alpha^H) \cdot \phi^H_{j,k} \qquad (6)$$

where the matrix M^H is called 'phantom response matrix'.

Now the phantom be replaced by a 'directionally dependent spectrometer', the most general type of detector considered in this paper. Its response $R_{J,K}$ is assumed to be distributed in two sets of discrete variables $J = 1,2...,J_m$ and $K = 1,2,...,K_m$. The variable J is related to the measurement of the radiation direction j, and may, for instance, be substantiated by turning the instrument to J_m different directions. The variable K is related to the measurement of the energy k of the radiation, and may, for instance, be substantiated by a pulse height spectrum. Even for fixed values of j and k, the measured values $R_{J,K}$ are allowed to be distributed. Simplifications introduced below will allow to consider less complicated detectors.

Defining corresponding quantities as in eqs. (3) and (5), the response of the detector can be written

$$R_{J,K} = \sum_{j=1}^{j_m} \sum_{k=1}^{k_m} M^R_{J,j,K,k} \cdot \phi^R_{j,k} \qquad (7)$$

The detector response matrix $M^R_{J,j,K,k}$ can be measured using 'monoenergetic' radiation with energy k and incident direction j. Inherent symmetries of usually employed detectors will in practice reduce the number of different matrix elements.

The inversion of eq. (7) is given by

$$\phi^R_{j,k} = \sum_{J'=1}^{J'_m} \sum_{K'=1}^{K'_m} P^R_{j,J',k,K'} \cdot R_{J',K'} \qquad (8)$$

Insertion of eq. (8) in eq. (7) gives

$$\sum_{j=1}^{j_m} \sum_{k=1}^{k_m} M^R_{J,j,K,k} \cdot P^R_{j,J',k,K'} = \delta_{J,J'} \cdot \delta_{K,K'} \qquad (9)$$

It is assumed that this system of linear equations can be used to express the projection matrix P by the measured response matrix M^R. A necessary condition is $J_m \cdot K_m \geq j_m \cdot k_m$ with $J_m = J_m'$ and $K_m = K_m'$.

It is further assumed that $M^H(\alpha-\alpha^H,E)$ and $M^R(\alpha-\alpha^R,E)$ have similar interpolation properties within the intervals. Then

$$\phi^H_{j,k} = \phi^R_{j,k} = \phi_{j,k} \tag{10}$$

The effective dose equivalent can be expressed by eqs. (6), (8) and (10):

$$H(\alpha^H) = \sum_{J=1}^{J_m} \sum_{K=1}^{K_m} M_{J,K}(\alpha^H) \cdot R_{J,K} \tag{11a}$$

where

$$M_{J,K}(\alpha^H) = \sum_{j=1}^{j_m} \sum_{k=1}^{k_m} M^H_{j,k}(\alpha^H) \cdot P^R_{j,J,k,K} \tag{11b}$$

As mentioned above, both matrices, M^H and P, can be considered to be known.

From eqs. (11a), (11b) the construction requirements can be deduced for four different types of detectors:

1. Requirement for the directionally independent dosimeter: M is independent of J and K, i.e.

$$H(\alpha^H) = M(\alpha^H) \cdot \sum_{J=1}^{J_m} \sum_{K=1}^{K_m} R_{J,K} = M(\alpha^H) \cdot R \tag{12}$$

The dosimeter only needs to indicate one single value R which is linearly proportional to H.

2. Requirement for the directionally dependent dosimeter: M is independent of K, i.e.

$$H(\alpha^H) = \sum_{J=1}^{J_m} M_J(\alpha^H) \cdot \sum_{K=1}^{K_m} R_{J,K} = \sum_{J=1}^{J_m} M_J(\alpha^H) \cdot R_J \tag{13}$$

This dosimeter only needs to indicate the values R_J.

3. Requirement for the directionally independent spectrometer: M is independent of J, i.e.

$$H(\alpha^H) = \sum_{K=1}^{K_m} M_K(\alpha^H) \cdot \sum_{J=1}^{J_m} R_{J,K} = \sum_{K=1}^{K_m} M_K(\alpha^H) \cdot R_K \tag{14}$$

This spectrometer only needs to indicate the values R_K.

4. The directional spectrometer, the most general case, has been discussed above. This instrument needs to indicate the complete information $R_{J,K}$ (see eq. (11a)).

OPERATIONAL QUANTITIES

In practice it may be advisable to simplify the construction requirements based on eqs. (11a), (11b).

In such cases a simplified phantom response matrix $\bar{M}^H$ can be determined such that

$$\bar{H}(\alpha^H) = \int_0^\infty dE \int_{2\pi} d\alpha \cdot \bar{M}^H(\alpha-\alpha^H, E) \cdot \phi_{E,\alpha}(\alpha, E) \qquad (15)$$

is an approximation to $H(\alpha^H)$. The quantity $\bar{H}$ with the corresponding response matrix $\bar{M}^H$ may facilitate the problem of measuring H, and it is defined as 'operational quantity'. Either one or both of the following simplifications can be introduced in order to construct an operational quantity:
(i) purely mathematical simplifications, for instance fitting the phantom response matrix to a simplifying analytical expression.
(ii) the phantom itself can be simplified and the corresponding response matrix be used.

DISCUSSION

It is concluded that Monte Carlo calculations for anthropomorphous phantoms are required as a prerequisite to construct the response matrix of the phantom. If, in addition, the response matrix of the detector is measured, then the effective dose equivalent can explicitly be expressed by the measured detector response. The extension of this concept to a three-dimensional distribution of distant sources does not pose any basic problems.

Regarding operational quantities, in the past, these often have been defined as the maximum of the spacial distribution of the 'dose equivalent index' [1,2], for which, however, a response matrix $\bar{M}^H$ (see eq. (15)) cannot be constructed because the corresponding superposition properties are lacking. Within the framework of the approach given here, such a quantity appears to be unsuitable.

ACKNOWLEDGEMENT - We like to thank Prof. Dr. S. Wagner for valuable discussions.

REFERENCES

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