ON THE POSSIBILITY OF THE DOSE FORMATION STUDY BY MEANS OF ČERENKOV RADIATION

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1. INTRODUCTION

For better insight into the processes of depth-dose distribution it is necessary to realize in detail the energy distributions of primary and secondary charged particles fluxes in the irradiated matter. As a rule, these distributions are determined theoretically, e.g. by solving of Boltzman transport equation or Monte-Carlo calculation. As regards the experimental data is hardly available since the existing methods are only adequate in the case of heavy particles, have a limited range of validity and a low resolution. For this reason it is hard to verify in detail the accuracy of the calculations and to obtain the independent information. It is suggested to obtain the energy distribution of particles on the intensity and angular distribution of Čerenkov radiation which is emitted from the small radiator inserted into the irradiated matter at the point of interest. It should be noted that the well known methods in high-energy physics based on the use of Čerenkov radiation allow to determine only the average velocity of fast charged particles for the monodirectional beam (1,2).

2. DESCRIPTION OF THE METHOD

A fairly thin plane-parallel Čerenkov radiator d thick is assumed to be located in the irradiated matter. Its position will be characterized by means of coordinate Z. Cartesian coordinate system $X'Y'Z'$ is introduced so that the plane $X'Y'$ is coincided with the front surface of the radiator and the $O'Z'$-axis is the normal to that surface. The direction of charged particles motion is given by polar and azimuthal angles ϑ_p and φ_p and the sense of photon emission is given by angles ϑ_R and φ_R. The distribution of particles on the front surface of radiator is assumed to be independent of coordinates $X'Y'$ and is given by function $f\,(Z'=0\,,\beta\,,\vartheta_p\,,\varphi_p)$, where β -is the velocity of the particles in units of speed of light .
The number of photons emitted with frequency ν in the interval $\Delta\nu$ per unit length traversed by a particle of charge z^*e is given by (3)

$$\frac{dN}{d\ell} = 2\pi\alpha z^{*2}\frac{\Delta\nu}{c}\left(1 - 1/n^2\beta^2\right) \qquad (1)$$

where $\alpha = \frac{1}{137}$ - is the fine structure constant;
n - is the absolute index of refraction.
This expression assumes that the medium is non-despersive, that is, the index of refraction is assumed to be constant over the frequency interval $\Delta\nu$ of emitted light. Then the number of photons emitted in the given direction within the

boundaries of the radiator is determined by the expression:

$$\frac{d}{d\Omega_R} N(n, Z, \vartheta_R, \varphi_R) = \frac{\Delta\nu}{c} \alpha d \int_{1/n}^{\beta_{max}} d\beta \left[1 - 1/n^2\beta^2\right] \int_0^{\pi} \sin\vartheta_p \, d\vartheta_p \int_0^{2\pi} d\varphi_p f(z'=0, \beta, \vartheta_p) \delta(\mu_0 - 1/\beta n) \quad (2)$$

where β_{max} is the maximum velocity of particles; $\mu_0 = \cos\Theta$; Θ - is the angle of photon emission with respect to the particle motion direction. The occurence of Dirac delta function reflects the fact that the particles emit the photons only at the strictly definite angle Θ , which is given by expression:

$$\Theta = \arccos(\beta n)^{-1} \quad (3).$$

Equation (2) has been derived on the assumption that the variation of distribution function within the boundaries of the radiator can be neglected.

For the calculation of dose distribution it is essential to know the energy distribution at the points of interest. Therefore we have confined ourselves to its obtaining, although the angular distribution of particles can be also found from the angular distribution and the value of Čerenkov flux. Integrating the expression (2) over all directions of photon emission we obtain the integral relation, which connects the total flux of Čerenkov radiation and the velocity distribution of particles

$$N(n, Z) = A \int_{1/n}^{\beta_{max}} \left(1 - 1/n^2\beta^2\right) f(Z'=0, \beta) \, d\beta \quad (4)$$

where

$$A = 2\pi z^2 \frac{\Delta\nu}{c} \alpha d; \quad f(z'=0, \beta) = \int_0^{2\pi} d\varphi_p \int_0^{\pi} \sin\vartheta_p \, f(z'=0, \beta, \vartheta_p, \varphi_p) \, d\vartheta_p$$

The expression (4) is the Volterra integral equation of first kind, where $N(n, Z)$ is the measured experimental value. The solution to this equation is

$$f(z'=0, \beta) = \frac{n'^2}{2A} \left[3 \frac{d}{dn} N(n', Z) + n' \frac{d^2}{dn^2} N(n', Z)\right] \quad (5)$$

where $n' = \frac{1}{\beta}$.

In the choice of the solution procedure it must be born in mind that the equation (4) is not well posed as with the most nuclear spectrometry problems.

The obtaining of the value $N(n, Z)$ resolves itself into the variation of n (e.g., ba replacement of radiator) and the shift of the radiator inside of irradiated matter. It should be noted that the integral equation (4) and its so-

lution (5) are correct also in the case, when the size of radiator is equal or greater than the lateral dimensions of irradiated region.
With the monodirectional particle beam of known direction of motion, the simple relation is obtained from expression (2), connecting the angular distribution of Čerenkov radiation and the velocity distribution of particles. If the normal to the surface of Čerenkov radiator is coincided with the direction of particle motion, i.e.

$$f(z'=0,\beta,\vartheta_P,\varphi_P) = B(\beta)\delta(\vartheta_P)\delta(\varphi_P)$$

then

$$B\left(\beta=\frac{1}{\mu_R n}\right) = \frac{T(n,\vartheta_R)n}{A\left(\frac{1}{\mu_R^2}-1\right)}\frac{d}{d\Omega_R}N(n,z,\vartheta_R,\varphi_R)$$

where $T(n, \vartheta_R)$ is a coefficient which takes into account the dependence of Čerenkov radiation flux emerged from the radiator on the angle of incidence on the boundary between the radiator and medium, $\mu_R = \cos\vartheta_R$

3. THE FIELD OF APPLICATION

As it was noted above, with the given function $f(Z,\beta)$, the dose formation process can be investigated in more detail. With electron or positron radiation (including secondary particles) the production threshold of Čerenkov radiation is reasonably low (e.g., for the medium with $n \div 2 \div 2{,}4$ it is about 60-÷50 KeV). This method allows for the energy distributions to be determined and the functional to be calculated from them over a wide range of energy. In the case of heavy charged particles these possibilities become somewhat fewer due to the more complicated interpretation of the results because of the secondary charged particle generation and the more high production thresholds of Čerenkov radiation.

This method is especially convenient for the high energy and intensive fluxes. This fact allows it to be used for the accelerator beam dosimetry, spectrometry and monitoring.

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