The Relation of β Radioactive Source Efficiency and Equivalent
Mass Attenuation Coefficient

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INTRODUCTION

The interaction of electronics and substance are of very important significance in many fields, like the nuclear physics, nuclear medical science, protection of nuclear, atmosphere physics and plasma physics etc all possesses. From the 20's this century, there are many science workers to engage in the aspect work \[1\], \[2\], \[3\], and a large number of syntheses discussing \[4\], \[5\], \[6\], \[9\] and experiment data \[7\], \[8\], \[10\], \[11\] and the model of theory calculation \[12\], \[13\] have been published.

In most of documents, the object researched is usually thin sources, and the source efficiency problem is neglected. Under considering the influence absorbing in the active source, this paper has gone on to this problem on theory and experiment.

RESEARCH METHOD

According of the document \[14\], the definition of the active response and the source efficiency follow:

2.1. Active Response
The active response is the count rate of every unit activity, showing s⁻¹Bq⁻¹.
Using the standard active source with A activity that is known, under one fixed geometry condition, the counting device gives the net counting rate \((n - n_b)\), then the active response of this counting device corresponding to the active source is:

\[
\varepsilon = \frac{n - n_b}{A}
\]

In the formula:
\(-\varepsilon\) - active response , \(A\) or \(s^{-1} \cdot Bq^{-1}\).
\(n\) - the counting rate of standard active source measured, \(s^{-1}\).
\(n_b\) - the background count rate of count device, \(s^{-1}\).
\(A\) - the activity of standard active source, \(Bq\).

2.2. Source Efficiency
Source efficiency is the specific rate value which the particle number of the superficial launching of active source \((q_{2\pi})\) is divided by producing or releasing particle number \((q_{4\pi})\) in the active source(thin source) or in saturated layer of thickness (thickness source) in unit time:

\[
\varepsilon_s = \frac{q_{2\pi}}{q_{4\pi}}
\]

In the formula:
\(-\varepsilon_s\) - source efficiency ;
\(q_{2\pi}\) - the superficial launching rate of active source (particle numbers of per second), \(s^{-1}\).
\(q_{4\pi}\) - total launching rate of active source, \(s^{-1}\).

2.3. Experimental Method
The source efficiency is concerned to the energy of the launching particle in a radioactive nuclide and a medium (like soil and living things sample ash and the water evaporating residues and various salts etc.) and a sample plate.
It uses the radioactive substance known quality activity to make certain thickness the sample source, and the sample’s surface launching rate is measured with \(2\pi\) flowing gas’s counting device. Then the source efficiency is obtained (see formula 2). In same kind of sample sources with different quality thickness, these source efficiencies are different. In same quality thickness samples, the source efficiencies of the different radioactive nuclide are also different. The source efficiency curve of different radioactive nuclide should generally be given by the experiment.
2.4. Theory Model

The condition

(1) $\beta$ radioactive nuclide is average distributed in the substance of source, and the active rate is $a$, Bq/mg.

(2) When its deflection angle is more than 90°, a $\beta$ particle is regarded that this $\beta$ particle is absorbed completely, namely the influence of many times scattering is neglected.

(3) The absorbing rule of $\beta$ particle approximately is obeyed the index attenuation law \cite{26} in the substance.

If $\mu_m$ is the equivalent quality attenuation coefficient, cm$^2$/mg; $X_m$ is the mass thickness of sample, mg/cm²; $q_0$ is the number that the $\beta$ particle is launched from the first surface in unit time when $x_m=0$, s$^{-1}$; $q_{x_m}$ is launching rate from the surface in $2\pi$ when the mass thickness is $x_m$, s$^{-1}$.

$$q_{x_m} = q_0 \cdot e^{-\mu_m x_m} \quad (3)$$

Shown as the figure 1, a kind of differential unit is selected in the sample source with the mass thickness $X_m$ and the area is $S$(cm²). Then the $\beta$ radioactive activity in the differential unit is $a \cdot (dx_m \cdot S)$.

![Diagram](image)

**Fig 1.** A diagrammatic sketch of the sample source

If the average number launching $\beta$ particle is $K_n$ in once decaying, then launching particle number along upward the $2\pi$ direction in the differential unit is:

$$\frac{1}{2} k_n \cdot a \cdot (dx_m \cdot S) = \frac{1}{2} k_n \cdot a \cdot S \cdot dx_m$$

If $X_m$ is total thickness of the sample (mg/cm²), the $\beta$ particle number launching out the source substance from the differential unit is:

$$dq_{x_m} = \left(\frac{1}{2} k_n \cdot a \cdot S \cdot dx_m\right) \cdot e^{-\mu_{\alpha}(X_m-x_m)}$$

$$= \frac{1}{2} k_n \cdot a \cdot S \cdot e^{-\mu_{\alpha}(X_m-x_m)} \cdot dx_m \quad (4)$$

If the formula (4) is integrated, then:

$$q_{2\pi} = \int_0^{2\pi} dq_{x_m} = \int_0^{X_m} \frac{1}{2} k_n \cdot a \cdot S \cdot e^{-\mu_{\alpha}(X_m-x_m)} \cdot dx_m$$

$$q_{2\pi} = \frac{1}{2} k_n \cdot a \cdot S \cdot \int_0^{X_m} e^{-\mu_{\alpha}x_m} \cdot dx_m$$

$$\therefore \quad q_{2\pi} = \frac{k_n \cdot a \cdot S \cdot X_m}{2\mu_{\alpha} \cdot X_m} \left(1 - e^{-\mu_{\alpha}x_m}\right) \quad (5)$$

According to the definition of source efficiency and $q_{4\pi} = k_n \cdot a \cdot S \cdot X_m$, then:

$$\varepsilon_s = \frac{q_{2\pi}}{q_{4\pi}} = \frac{1}{k_n \cdot a \cdot S \cdot X_m} \cdot \frac{k_n \cdot a \cdot S \cdot X_m}{2\mu_{\alpha} \cdot X_m} \left(1 - e^{-\mu_{\alpha}x_m}\right)$$

$$\varepsilon_s = \frac{1}{2} \cdot \frac{1 - e^{-\mu_{\alpha}x_m}}{\mu_{\alpha} \cdot X_m} \quad (6)$$
After the influence that the factor is not determined in the experiment is considered, the constants of the formula (6) are changed to the following constant k and c, which will be treated to fix. See the formula (7).

\[
\varepsilon_s = k \cdot \frac{1 - e^{-\mu_m X_m}}{\mu_m X_m} + c \quad (7)
\]

The relation of source efficiency, equivalent mass attenuation coefficient \(\mu_m\) and mass thickness \(X_m\) of the sample source has been established in the formula (7).

**DATA PROCESSING**

3.1. The experiment data:

After the sample source with the certain thickness is made by the radioactive substance of known the active rate, the surface launching rate of sample source is measured in \(2\pi\) following gas type measuring system. After the rate of the surface launching rate and total activity for sample source is calculated according to formula (2), the source efficiency is given for certain one thickness sample.

Having measured respectively nuclides such as \(^{147}\text{Pm},^{60}\text{Co},^{137}\text{Cs},^{40}\text{K}\) and \(^{90}\text{Sr}^{90}\text{Y}\) etc with the 50mm\(^2\) stainless steel sample plate, we get the many source efficiencies for different mass thickness. The figure 2 is the relation of the source efficiency and the mass thickness for several kinds of nuclide according to experiment data.

![Fig. 2 The source efficiency of B radioactive nuclide](image)

3.2. The data fitting

Figure 3~7 gives respectively the relation curve of source efficiency of \(^{147}\text{Pm},^{60}\text{Co},^{137}\text{Cs},^{40}\text{K}\) and \(^{90}\text{Sr}^{90}\text{Y}\) radioactive nuclide and the sample mass thickness. The values are that calculates to fit in the figures according to the formula (7). That the table 1 gives is respectively the fitting parameter for survey nuclides.

<table>
<thead>
<tr>
<th>Nuclide</th>
<th>k</th>
<th>(\mu_m) (cm(^2)/mg)</th>
<th>c</th>
</tr>
</thead>
<tbody>
<tr>
<td>(^{147}\text{Pm})</td>
<td>0.423</td>
<td>0.203</td>
<td>0</td>
</tr>
<tr>
<td>(^{60}\text{Co})</td>
<td>0.347</td>
<td>0.982</td>
<td>0.0158</td>
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<tr>
<td>(^{137}\text{Cs})</td>
<td>0.423</td>
<td>0.0395</td>
<td>0.0153</td>
</tr>
<tr>
<td>(^{90}\text{Sr}^{90}\text{Y})</td>
<td>0.556</td>
<td>0.0263</td>
<td>0.0123</td>
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<td>(^{40}\text{K})</td>
<td>0.566</td>
<td>0.0142</td>
<td>0.0528</td>
</tr>
</tbody>
</table>
Fig. 3 The relation of $^{147}$Pm source efficiency and sample mass thickness

Fig. 4 The relation of $^{60}$Co source efficiency and sample mass thickness

Fig. 5 The relation of $^{137}$Cs source efficiency and sample mass thickness
DISCUSS

4.1. The results of figure 3~7 indicate that the experiment value has very good identical property with the calculation value. Thus, we may think it correct that theory hypothesis and derivation conclusion ahead. They have universal significance.

4.2. The corresponding value can be got, when $\mu_m$ is known according to the formula (7).

4.3. When the coefficient $k$ and $c$ of the formula (7) is unknown, they must generally be determined by use of the experiment. But the theory values may be use to estimate when the condition is not allowed.

Namely: $k = 0.5$, $c = 0$.

4.4. According to demonstration course ahead and the document [15], it is known that the above-mentioned conclusion is also approximately fit for the fission product. At this moment, the source efficiency only need be substituted with the concept of equivalent source efficiency.

REFERENCES